UNIVERSITY OF MANITOBA
Term Test 3D
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 4 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 50 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	4	
2	2	
3	7	
4	6	
5	9	
6	22	
Total:	50	

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[4] 1. Let $h(x) = (3f(x) + 2)(g(x) + x)$, Find $h'(1)$ if $f'(1) = g'(1) = 2$ and $f(1) = g(1) = -2$.

Solution:
Using the product rule for derivatives,

$$\begin{aligned} h'(x) &= (3f(x) + 2)'(g(x) + x) + (3f(x) + 2)(g(x) + x)' \\ &= 3f'(x)(g(x) + x) + (3f(x) + 2)(g'(x) + 1) \end{aligned}$$

Therefore,

$$\begin{aligned} h'(1) &= 3f'(1)(g(1) + 1) + (3f(1) + 2)(g'(1) + 1) \\ &= 3(2)(-2 + 1) + (3(-2) + 2)(2 + 1) = -18 \end{aligned}$$

[2] 2. Given that, for a function $f(x)$, $f''(x)$ is continuous near $x = c$, $f'(c) = 0$, and $f''(c) < 0$, what can we conclude about $f(c)$ in terms of relative (local) extrema? Explain your answer.

Solution: Since $f'(c) = 0$, f is defined at $x = c$ and has a critical number at $x = c$. Since $f''(c) < 0$, according to the second derivative test, f has a relative (local) maximum at $x = c$.

[7] 3. Find the absolute extrema of $f(x) = -x^3 + 12x$ on the interval $[-3, 5]$. Justify your answer.

Solution: Since f is a polynomial function, it is continuous on the closed interval $[-3, 5]$.

$$f'(x) = -3x^2 + 12$$

which is defined everywhere.
To find critical numbers, we have to solve $f'(x) = 0$.

$$f'(x) = 0 \Rightarrow -3x^2 + 12 = 0 \Rightarrow -3(x^2 - 4) = 0 \Rightarrow x = -2 \text{ or } x = 2.$$

Hence, with the endpoints, we test $x = -2$ and $x = 2$, since they are both in the interval.

$$\begin{aligned} f(-2) &= -16 \\ f(2) &= 16 \\ f(-3) &= -9 \\ f(5) &= -65. \end{aligned}$$

Then, the absolute maximum is 16 and the absolute minimum is -65.

[6] 4. Suppose that $f(x)$ is differentiable on an interval I and $f'(x) > 0$. Show that $f(x)$ is increasing on I.

Solution: Let x_1 and x_2 be any two distincts numbers in I such that $x_2 > x_1$. Since $f'(x)$ exists on I, $f(x)$ is differentiable on I and thus it is continuous on I. Therefore, we have that $f(x)$ is continuous on $[x_1, x_2]$ and differentiable on (x_1, x_2).
So by the Mean Value Theorem, there exists c between x_1 and x_2 such that

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1).$$

Since $f'(c) > 0$ and $x_2 - x_1 > 0$, we conclude that $f(x_2) - f(x_1) > 0$ and, hence, $f(x_2) > f(x_1)$. Thus, $f(x)$ is increasing on I.

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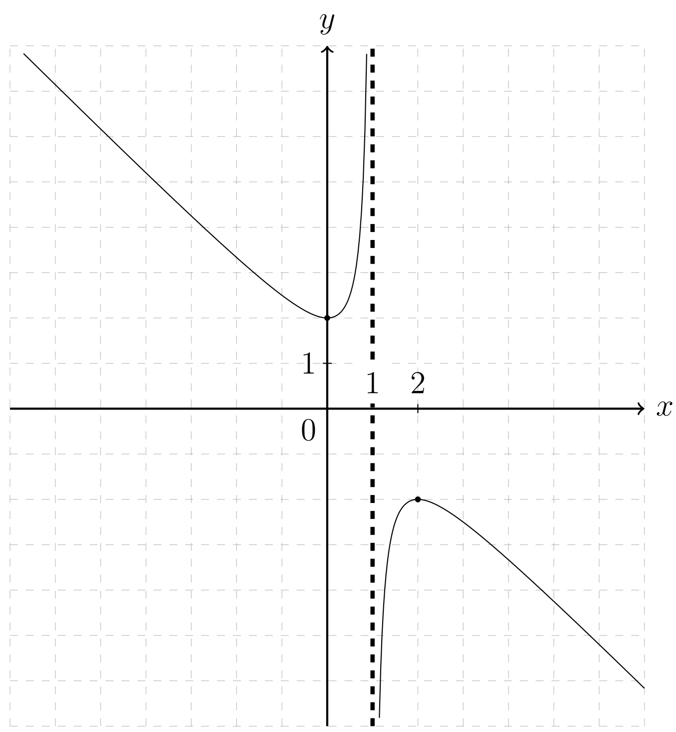
5. The graph of the function $f(x) = \dfrac{-x^2+2x-2}{x-1}$ is given below. Use this graph to gather information and fill in the blanks. (*If a feature doesn't apply, write "None."*)



[1] (a) Equation(s) of any vertical asymptotes: $x = 1$

[1] (b) Equation(s) of any horizontal asymptotes: None

[1] (c) Open intervals where f is increasing: $(0, 1)$, $(1, 2)$

[1] (d) Open intervals where f is decreasing: $(-\infty, 0)$, $(2, \infty)$

[1] (e) x and y-coordinates of any local maxima: $(2, -2)$

[1] (f) x and y-coordinates of any local minima: $(0, 2)$

[1] (g) Open intervals where f is concave up: $(-\infty, 1)$

[1] (h) Open intervals where f is concave down: $(1, \infty)$

[1] (i) x and y-coordinates of any inflection point(s): none

6. Use the function $f(x)$, the first derivative $f'(x)$ and the second derivative $f''(x)$ as defined here to gather information and fill in the blanks below. (*If a feature doesn't apply, write "None."*)

$$f(x) = \frac{x^2}{4-x^2} \qquad f'(x) = \frac{8x}{(4-x^2)^2} \qquad f''(x) = \frac{8(3x^2+4)}{(4-x^2)^3}$$

[1] (a) Domain of f: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

[1] (b) Symmetry of f: Even

[1] (c) x-intercepts: 0

[1] (d) y-intercept: 0

[1] (e) Equation(s) of any vertical asymptotes: $x=-2, x=2$

[1] (f) Equation(s) of any horizontal asymptotes: $y=-1$

[1] (g) x and y-coordinates of any critical point(s): $(0,0)$

[2] (h) Open intervals where f is increasing: $(0,2)$, $(2,\infty)$

[2] (i) Open intervals where f is decreasing: $(-\infty,-2)$, $(-2,0)$

[1] (j) x and y-coordinates of any local maxima: None

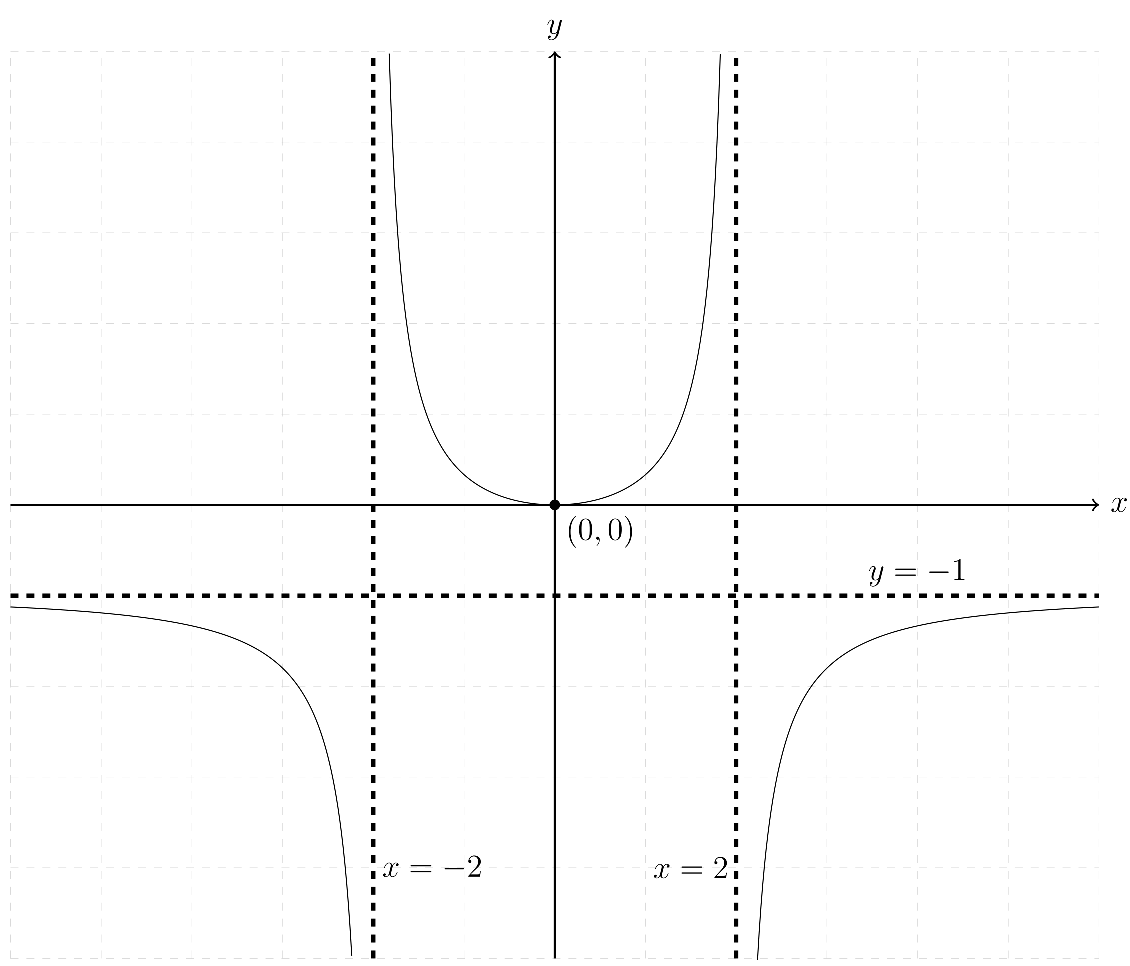
[1] (k) x and y-coordinates of any local minima: $(0,0)$

[2] (l) Open intervals where f is concave up: $(-2,2)$

[2] (m) Open intervals where f is concave down: $(-\infty,-2)$, $(2,\infty)$

[1] (n) x and y-coordinates of any inflection point(s): None

[4] (o) Use the information from the previous parts to give a neat sketch of the graph $y=f(x)$, making sure that you label all important features of the graph.



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If you are using this page to continue your work from a previous question, clearly indicate on the original page that your work is continuing here. Otherwise, your work will not be marked.

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